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An Analytical Method for Static Earth Pressure Distribution against Rectangular Shallow Tunnels Using Lateral Deformation

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ABSTRACT
Analytical methods for computing the lateral earth pressure against tunnel is vastly used by engineers all over the world. Conventional analytical methods compute the lateral pressure in either active or passive state while the stress state usually falls between these two boundaries in many practical cases. Furthermore, using these boundary coefficients lead to either overestimated or underestimated results in design. Thus, a modified method based on the strain increment theory for calculating the lateral pressure against rectangular tunnels is presented herein to consider the amount of lateral deformation at each depth. First, the results for different values of overburden depth, friction angle and wall mobilized angle are investigated. Then comparative finite element analyses were performed to examine the effectiveness of the method. According to this study, the pressure pattern is completely nonlinear especially at the corners of tunnel lining. In fact, the pressure increases nonlinearly to about three times of the value at top. Lateral earth pressure decreases with the increase of friction angle which is in good agreement with finite element results. Overall, the pressure patterns derived by this method for shallow depths (less than tunnel height) are almost the same as those computed by finite element method.
Keywords-Cut-and-cover, earth pressure, Finite element, strain increment method, tunnel

I. INTRODUCTION

Nonetheless, various computer software using numerical methods has been developed in the recent decades to analyze underground structures and to derive the induced forces, the use of analytical methods to compute the earth pressure directly is very popular among engineers. They often employ either Rankine [1] or Coulomb [2] relationships for determining the earth pressure at active and passive states. Moreover, Jaky [3]'s equation is widely used when the value of earth pressure at rest is desired.

Zhang et al. [4] presented an innovative theoretical method for deriving lateral earth pressure against a rigid wall under any amount of horizontal displacement and the mode of deformation by introducing a new parameter called lateral strain ratio (R). The purpose of this paper is to extend this new concept to rectangular cut-and-cover tunnels as well. To achieve this goal, lateral earth pressure distribution against a boxed-shape cut-and-cover tunnel is computed by strain increment method (SIM) and the results are compared with a consistent finite element (F.E.) analysis to investigate the effectiveness of this method. This study is part of a research line in Curtin University and the dynamic modelling of the lateral pressure has been conducted previously [5].

II. METHODOLOGY

Zhang et al. [4] conducted a series of experimental tests and found there is a relation between lateral earth pressure coefficient (*K*) and a new lateral strain parameter (*R*) for any amount of lateral deformation (Δ) in normally consolidated cohesionless soils:

$$R = \begin{cases} -\left(\frac{|\Delta|}{\Delta_a}\right)^a, & -\Delta_a \leq \Delta \leq 0 \\ -1, & \Delta < -\Delta_a \end{cases} \quad (1)$$

$$R = \begin{cases} 3\left(\frac{\Delta}{\Delta_p}\right)^p, & 0 \leq \Delta \leq \Delta_p \\ 3, & \Delta > \Delta_p \end{cases} \quad (2)$$

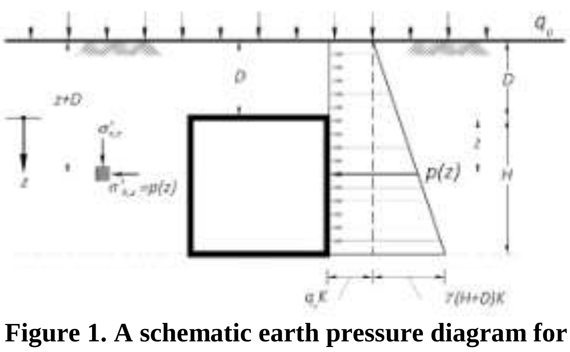

Figure 1. A schematic earth pressure diagram for shallow tunnel

where Δ_a and Δ_p are the minimum values of deformation required to develop active and passive states and can be computed by experimental relationships [6-8]. Zhang et al. [4] recommended 0.3 to 0.5 for *a* and 0.4 to 0.5 for *p*.

The deformation mode of cut-and-cover tunnel can be derived experimentally or similarly by the following non-uniform equation[4, 9]:

$$\frac{\Delta}{\Delta_{max}} = -\frac{\left(1-\frac{z}{H}\right)^m \left(\frac{z}{H}\right)^n}{\left(1-\frac{z_m}{H}\right)^m \left(\frac{z_m}{H}\right)^n} \qquad (3)$$

where m and n are the exponents to fit the curve best to the real deformation of the wall and z_m is the depth at which max displacement happens. In this research, various values for m and n are used and the values equal 4 developed the best fit for the side wall deformation.

Zhang et al. [4] developed new equations for calculating lateral earth pressure coefficient for any intermediate state of stress based on the well-known formulations of Rankine and Coulomb. Equation (4) and (5) give lateral earth pressure coefficient (K) based on extending Rankine formulation. Similarly, Coulomb-based lateral earth pressure coefficient (K') can be derived from equations (6) and (7):

$$K = \frac{1-\sin\phi'}{1-\sin\phi' R} \quad , -1 \le R \le 1 \qquad (4)$$

$$K = 1 + \frac{\sin\phi'}{1-\sin\phi'}(R-1) \quad , 1 \le R \le 3 \qquad (5)$$

$$K' = \frac{2\cos^2\phi'}{\cos^2\phi'(1+R) + \cos\delta_{mob}(1-R)\left[1+\sqrt{\frac{\sin(\phi'+\delta_{mob})\sin\phi'}{\cos\delta_{mob}}}\right]^2} \quad , -1 \le R \le 1 \qquad (6)$$

$$K' = 1 + \frac{1}{2}(R-1)\left(\frac{\cos^2\phi'}{\cos\delta_{mob}\left[1-\sqrt{\frac{\sin(\phi'+\delta_{mob})\sin\phi'}{\cos\delta_{mob}}}\right]^2} - 1\right) \quad , 1 \le R \le 3 \qquad (7)$$

where ϕ' and δ_{mob} are the effective friction angle of soil and the mobilized friction angle of wall respectively. Fig. 1 shows a shallow rectangular tunnel and the lateral earth pressure distribution against the side walls. Since the tunnel is buried in a shallow depth, the overburden depth has been considered as part of the surcharge in the calculation of the lateral earth pressure in this research. By using SIM, lateral earth pressure at the depth of z can be computed from equations (4) to (8):

$$p(z) = \sigma'_{h,z} = (q_0 + \gamma D + \gamma z)K \quad 0 \le z \le H \qquad (8)$$

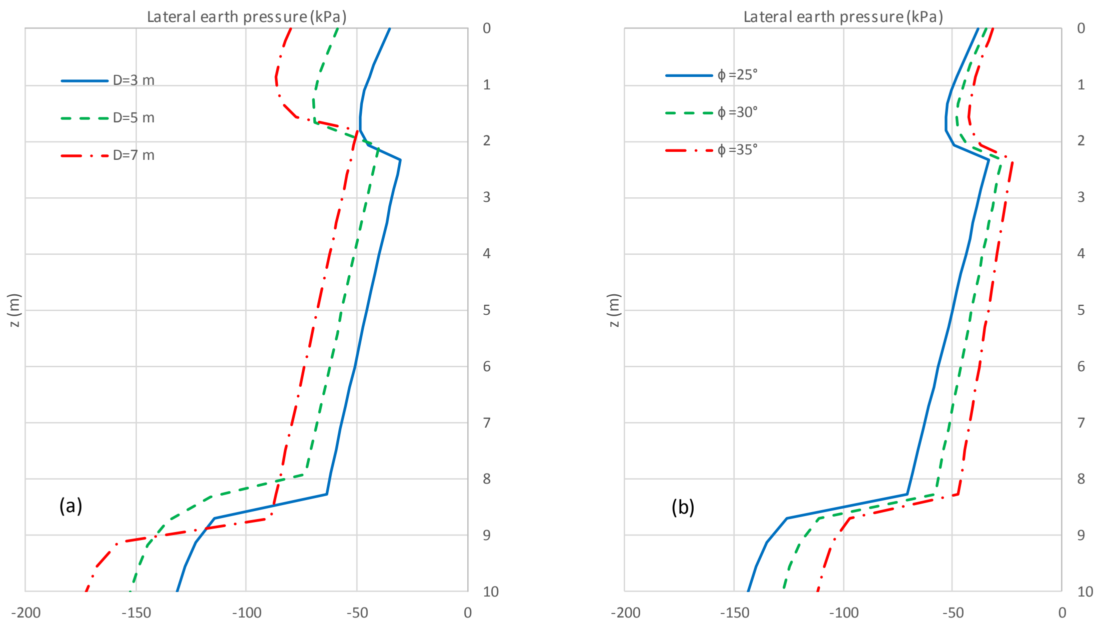

Figure 2. Lateral earth pressure distribution a) Depth variation b) ϕ' variation

where q_0 is the surcharge load (kN/m^2), γ is the soil unit weight (kN/m^3), D is the overburden depth (m), H is the height of tunnel (m) and K is the lateral earth pressure coefficient derived by SIM.

III. RESULTS AND DISCUSSION

According to SIM concept presented in the previous sections for shallow tunnels, lateral earth pressure distribution against the side walls of a box-shaped tunnel is computed for various values of overburden depth (*D*), frictional angle (ϕ') and mobilized wall friction angle (δ_{mob}) in this section and the effect of each aforementioned parameter is investigated in comparison with the related finite element results performed by PLAXIS® [10]. The studied tunnel is a 10-m square section which is buried at the depth of *D*. A cohesionless homogenous soil (sand) is taken into account for comparative analyses to be able to employ the strain increment theory. The other geotechnical parameters for the studied soil are: Soil unit weight (γ)=17kN/m^3, frictional angle (ϕ')=25°, 30° and 35°, Young modulus (*E*)=130MPa, Poisson's ratio=0.3, and mobilized wall frictional angle (δ_{mob})=0.5$\phi'$and 0.67ϕ'.

The first step in computing the lateral earth pressure using SIM, is to determine the lateral deformation of the side walls of tunnel. It can be derived simply by equation (3) or any other prescribed deformation. Then by using equations (1) and (2), lateral strain ratio (*R*) is calculated at each particular depth (*z*). In this research, the values of active and passive displacements are: Δ_a=0.005H, and Δ_p=0.014H. Finally, lateral earth pressure can be derived from equations (4) to (8) for Rankine-based SIM or Coulomb-based SIM relationships depending on the value of *R*at each depth of tunnel height.

Fig.2(a) shows the lateral earth pressure distribution against the sidewalls of tunnel for ϕ'= 30° and δ_{mob} =0.5ϕ' for differnt depth of overburden (*D*). As shown in this figure, for *D=3m*, lateral earth pressure (*p*) increases nonlinearly from 35 kPa at *z=0* (top of lining) to its peak value, 49 kPa at *z=1.8m*, followed by an abrupt decrease to 30 kPa at *z=2.3m*. Then it gradually grows to 64 kPa at *z=8.3m*. Afterwards, *p* continues to grow with a greater rate and reaches 123 kPa at almost 1m from the bottom of lining. This sharp increase followed by a slight increase at the bottom of lining reaches the value of 131 kPa. Despite linear behaviour at the middle part of the tunnel sidewalls, the pressure distribution is totally nonlinear at both top and bottom parts.

Lateral earth pressure at two other overburden depths of 5m and 7m are presented in this figure as well. As seen, the patterns of pressure distribution for these depths are very similar to the 3m graph. However, the value of pressure increases with the increase in *D* especially at the middle height of the tunnel wall.

Fig.2(b) shows the pressure distribution for different friction angles (*D*=3m and δ_{mob} =0.5ϕ'). As seen in this figure, the lateral earth pressure computed by SIM decreases with increasing ϕ'.

III.1.Frictional angle (ϕ') variation

Fig.3 shows the lateral earth pressure distribution against the sidewalls of tunnel with the variation of friction angle (ϕ'), for overburden depth of 3 m and $\delta_{mob} = 0.5\phi'$. In this figure, the lateral earth pressure (*p*) computed by both Rankine- and Coulomb-based SIM and the consistent F.E. results are presented. In all figures, SIM pressure distribution follows the same pattern of F.E. result. However, the value of Coulomb-based SIM pressure is less than Rankine-based SIM pressure because of considering the wall friction angle. According to SIM, lateral earth pressure distribution is completely nonlinear at both top and bottom corners of the box shaped tunnel (i.e. z=0 to 2.5 m at top and z=8 to 10 m at bottom) which is in good agreement with the F.E. result. Furthermore, the lateral earth pressure pattern in the middle height of the tunnel wall is linear which is almost compatible with the F.E. pattern as well. Since SIM is not able to consider stress concentration at the corners, SIM results are slightly overestimated and underestimated in the middle and corners (either at top or bottom of lining) respectively. This difference between SIM and F.E. values due to stress concentration grows higher at the corners with the increase in the friction angle.

III.2.Mobilized wall friction angle (δ_{mob}) variation

The effect of interface condition on the results of SIM is presented in this section. Two different values of δ_{mob} (equals to 0.5ϕ', and 0.67ϕ') are taken into account for the wall friction angle. To investigate this effect, several combination of overburden depth and friction angle are used in this study. Yet only the results of overburden depth of 3 m and friction angle (ϕ') of 30° is presented here in Fig.4. As seen in these graphs, SIM follows the same pattern for lateral earth pressure distribution for all different values of wall friction angleδ_{mob}. However, because of stress concentration at the corners of box-shaped lining, the difference between earth pressures computed by these two methods is getting higher at the top and bottom of lining.

III.3. Overburden depth (D) variation

Fig.5shows the effect of overburden depth (*D*) on the results computed by SIM for both Rankine- and Coulomb-based procedure. Different values for friction angle (ϕ' =25°, 30° and 35°) and wall friction angle (δ_{mob} =0.5ϕ', and 0.67ϕ') are considered to evaluate the effect of overburden depth on the results. Briefly, only the results for ϕ' =30° and δ_{mob} =0.5ϕ' are presented here. As seen in this figure, the lateral earth pressures computed by SIM are compatible with the F.E. results in all different depths. However, the difference between SIM and F.E. values is getting higher with the increase in overburden depths because of the assumption made in the extended SIM equations for shallow tunnels. In these formulas, the effect of overburden depth is considered as part of surcharge. Therefore, the application of extended SIM for box-shaped cut-and-cover tunnels shall be limited to only shallow depths.

IV. CONCLUSIONS

In this paper a modified procedure based on strain increment method was proposed for calculating the lateral earth pressure distribution against side walls of a shallow box-shaped tunnel. For the studied cases in this study, strain increment method (SIM) shows a good compatibility with the finite element (F.E.) results. SIM can also predict the non-linear behaviour of lateral earth pressure (*p*) at the corners of the box-shaped tunnel.

For overburden depth of 3m, and ϕ' =30°, *p* increases nonlinearly from 35kPa to three times of therelated value at top of the lining in both F.E. and SIM results. The value of *p* computed by SIM Coulomb-based formulation is less than the related value by Rankine-based formulation due to considering the effect of wall friction. The lateral earth pressure decreases with the increase of friction angle in both F.E. and SIM. However, the difference between SIM and F.E. values is higher at the corners of tunnel lining because of stress concentration.

The results state that SIM is more accurate in predicting lateral earth pressure for shallower depth. Therefore, the applicability of SIM shall be limited to only cut-and-cover tunnels buried at shallow depth which the assumption of considering the overburden depth as surcharge in formulation derivation is acceptable.

Further research including experimental laboratory tests shall be performed to evaluate the reliability of SIM before implementing in engineering practice.

V. ACKNOWLEDGEMENT

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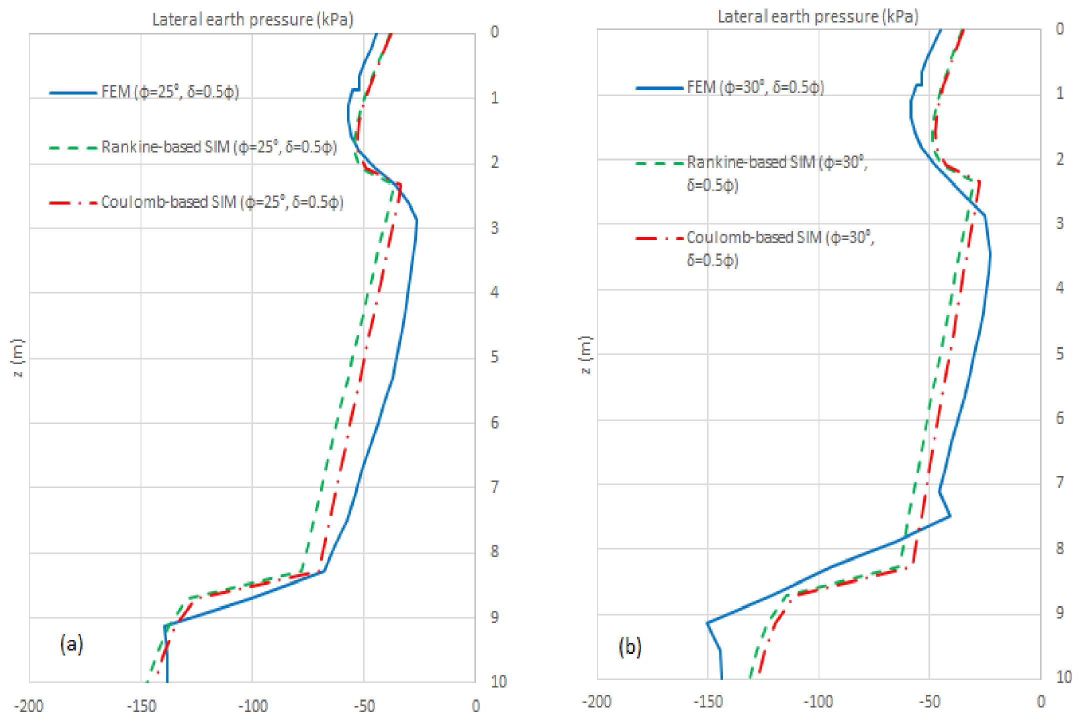

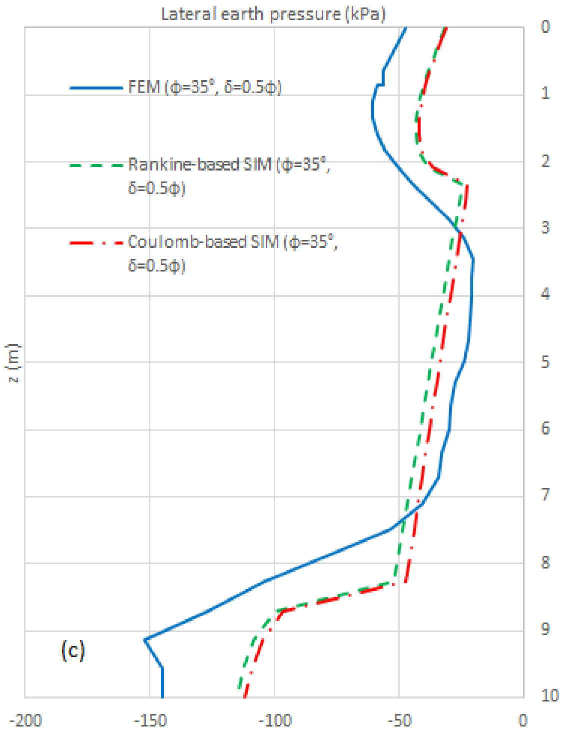

Figure 3. Variation of lateral earth pressure with ϕ'

a) ϕ'=25°
b) ϕ'=30°
c) ϕ'=35°

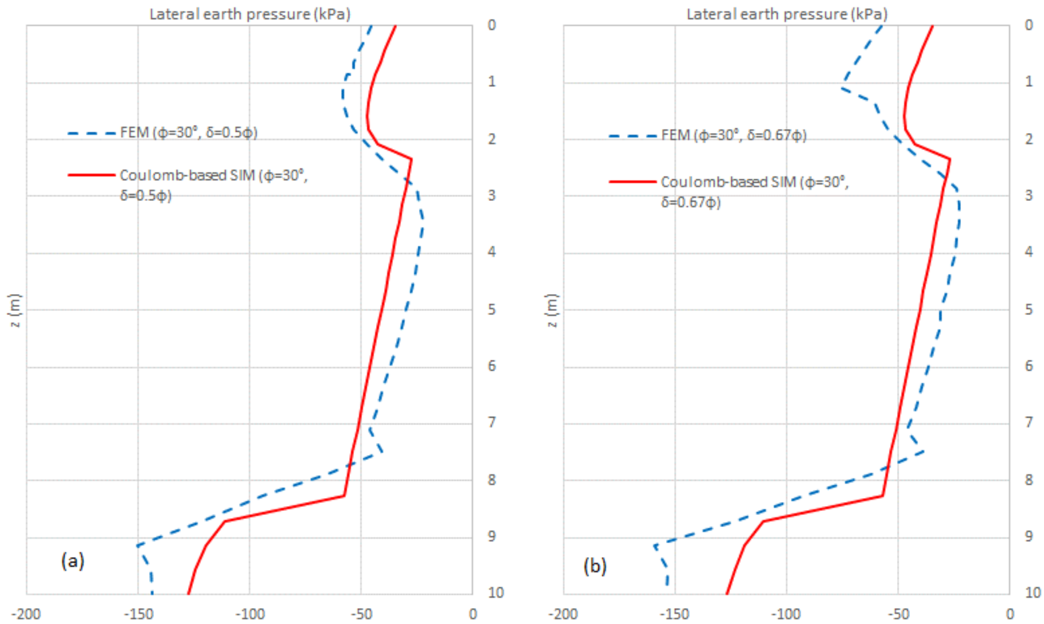

Figure 4. Variation of lateral earth pressure with δ

a) δ=0.5ϕ'
b) δ=0.67ϕ'

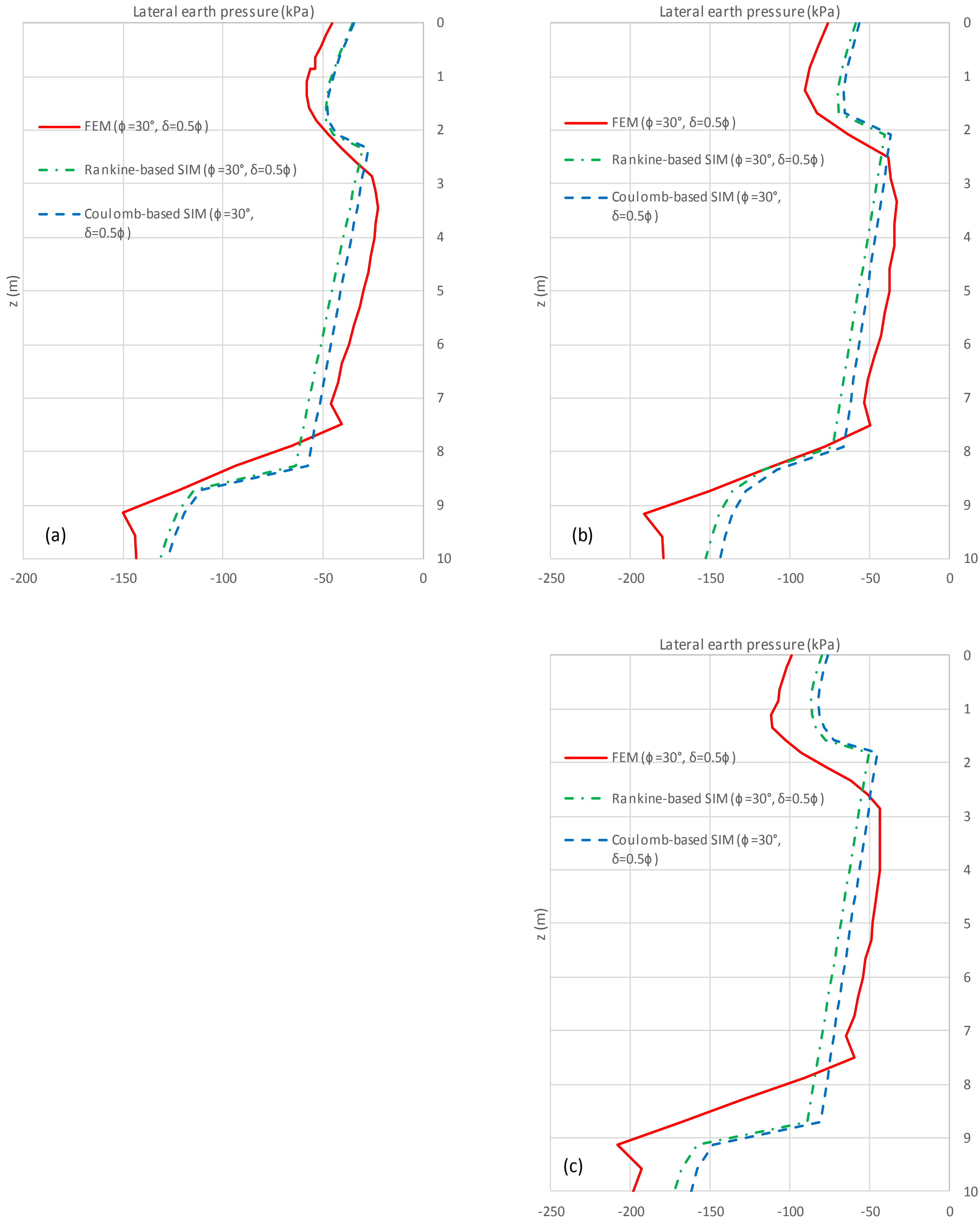

Figure 5. Variation of lateral earth pressure with overburden depth

a) D=3 m
b) D=5 m
c) D=7 m

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